NUMERICAL SOLUTION OF THE PROBLEM OF THE FLOW OF A SUPERSONIC GAS STREAM OVER THE UPPER SURFACE OF A DELTA WING IN THE EXPANSION REGION

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A numerical method is described for the calculation of supersonic flow over the arbitrary upper surface of a delta wing in the expansion region. The shock wave must be attached everywhere to the leading edge of this wing from the side of the lower surface. The stream flowing over the wing is assumed to be nonviscous. A problem with initial conditions at some plane and with boundary conditions at the wing surface and the characteristic surface is set up for the nonlinear system of equations of gas dynamics. The difference system of equations, which approximates the original system of differential equations on a grid, has a second order of accuracy and is solved by the iteration system proposed in [1]. The initial conditions are determined by the method of establishment of self-similar flow. A number of examples are considered. Comparison is made with the solutions of other authors and with experiment.

1. Let us examine the supersonic flow over a delta wing, assuming that the component of the velocity vector of the impinging stream normal to the leading edge is greater than the speed of sound and the bow shock wave is attached to the leading edge of the wing. The flows above and below the wing do not affect one another and can be examined separately. A solution of the problem for compression flow was given in [2]. Let us consider the expansion flow which develops at the upper surface of the wing if the angle of attack of the wing computed in the plane normal to the leading edge becomes greater than half the angle of the nose cross section in the same plane. The region of flow will be bounded by the wing surface and the characteristic surface emerging from the leading edge. We will assume that the surface of the wing is arbitrary. If it is conical the flow at the upper surface in the indicated region will possess the self-similar properties of conical flow.

Let us introduce a Cartesian coordinate system with the origin at the tip of the wing. The x axis is located in the vertical plane of symmetry, the z axis is directed along the wing span to the left, and the y axis is directed upward. The velocity vector of the impinging stream has an arbitrary angle of attack and lies in the xy plane (Fig. 1). The latter condition may be excluded and then the flow will be accompanied by slippage. The introduction of the condition of symmetry of the flow is done to reduce the calculations.

We will assume that the impinging stream is nonviscous and non-heat-conducting. We will divide the region of flow into three parts with the plane Q_0 and the surface Q_1. The plane Q_0 coincides with the plane $x = \text{const}$ and the surface Q_1 coincides with the surface $\eta = \text{const}$, where $\eta = z/H(t)$ and $H = H(t)$ is the equation of the leading edge of the wing (see Fig. 1).

The solution of the problem is divided into the three problems of determining the flow in regions 1, 2, and 3. The solution of the problem for region 1 gives the initial conditions in the plane Q_0 and the solution for region 2 gives the boundary values at the surface Q_1. Assuming that the problems for regions 1 and 2 are solved we can formulate the boundary problem for region 3 and give an algorithm for its numerical solution, and then give algorithms for the problems in regions 1 and 2.

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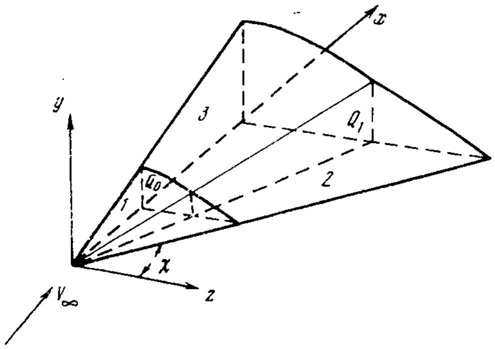

Fig. 1

Let us write the equation of continuity of flow, energy, and motion in matrix form:

$$A_1 \frac{\partial X}{\partial x} + B_1 \frac{\partial X}{\partial y} + C_1 \frac{\partial X}{\partial z} = 0 \tag{1.1}$$

Here A_1, B_1, and C_1 are square matrices of fifth order with components a_{ij}, b_{ij}, and c_{ij}:

$$a_{11} = b_{12} = c_{13} = \rho, \qquad a_{21} = b_{22} = c_{23} = \rho a^2,$$
$$a_{51} = a_{42} = a_{33} = a_{24} = a_{15} = u$$
$$b_{51} = b_{42} = b_{33} = b_{24} = b_{15} = v, \; c_{51} = c_{42} = c_{33} = c_{24} = c_{15} = w$$
$$a_{54} = b_{44} = c_{34} = 1 / \rho$$

and the other components of the matrices are zero; X is the vector column with components u, v, w, p, ρ; u, v, and w are the components of the velocity vector along the x, y, and z axes, respectively, relative to $\sqrt{p_\infty/\rho_\infty}$; p is the pressure relative to p_∞, and ρ is the density relative to ρ_∞; $a^2 = k_\infty p/\rho$; k_∞ is the ratio of specific heat capacities.

Let y = G(x, z) be the equation of the upper surface of the wing and y = F(x, z) be the equation of the outer characteristic surface. At the surface of the wing the condition of nonflow is correct. At the characteristic surface the vector X, which has the same components as in the impinging stream, is known while the function F(x, z) is unknown.

The boundary problem for region 3 is formulated as follows: the vector X and the function F(x, z) are known at the plane Q_0 and the surface Q_1. The solution of system (1.1) must be found in this region with the boundary conditions at the plane Q_0 and the surface Q_1 given at the surface of the wing and at the characteristic surface and with symmetry conditions at the plane of symmetry.

The solution of this problem is constructed numerically. First the vector X and the function F are found in a plane $Q_0^{(1)}$ close to Q_0, then the plane $Q_0^{(1)}$ is taken as the reference plane, and the process is repeated up to the trailing edge of the wing.

Let us convert to new coordinates in Eqs. (1.1) so that region 3 of the solution has the form of a parallelepiped:

$$x \to t, \quad y \to \xi = \frac{y - G}{F - G}, \quad z \to \eta = \frac{z}{H(t)}$$

In these coordinates the region of the solution will be characterized by the inequalities $t > t_0$, $0 \le \xi \le 1$, and $0 \le \eta < 1$ and the system of equations

$$A \frac{\partial X}{\partial t} + B \frac{\partial X}{\partial \xi} + C \frac{\partial X}{\partial \eta} = 0$$
$$A = A_1, \quad B = A_1 \xi_x + B_1 \xi_y + C_1 \xi_z, \quad C = A_1 \eta_x + C_1 \eta_z$$
$$\xi_x = -\frac{1}{(F-G)H} \{H[(1-\xi)G_t + \xi F_t] - \eta H_t [(1-\xi)G_\eta + \xi F_\eta]\} \tag{1.2}$$
$$\xi_y = \frac{1}{F-G}, \quad \xi_z = -\frac{1}{(F-G)H} [(1-\xi)G_\eta + \xi F_\eta]$$
$$\eta_x = -\eta H_t / H, \quad \eta_y = 0, \quad \eta_z = 1 / H$$

The algorithm for the numerical solution of the problem for region 3 coincides with the algorithm for the solution of the problem for compression flow [2] with the difference that the necessity of determining the vector X at the outer boundary of the region falls out. A rectangular grid is constructed in the desired region and an implicit difference system of second-order accuracy is used. The solution of the system of difference equations is carried out from layer to layer by iteration. The algorithm for this process has been presented in [1-3].

2. The problem for region 2 is solved on the assumption that the stream flows over the leading edge as a slipping wedge and in the plane normal to the leading edge the flow obeys the Prandtl–Meyer law. The gas-dynamic functions are constant along rays emerging from the leading edge of the wing in this plane and depend only on the angle between a given ray and the horizontal plane. This angle is a function of the coordinate ξ.

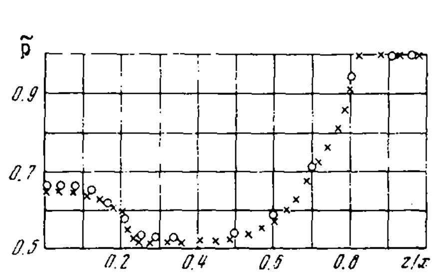

Fig. 2

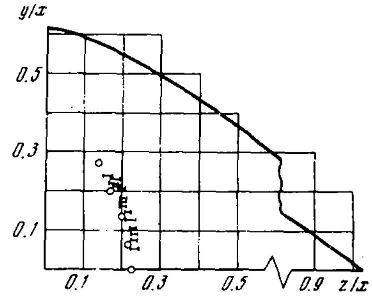

Fig. 3

To calculate the gasdynamic functions along the rays and consequently at the surface Q_1 which they intersect, the velocity vector of the impinging stream is resolved into two components. One of them (V_{τ_∞}) is directed along the tangent to the leading edge. It remains constant. The other (V_{n_∞}) lies in the plane normal to the leading edge. In the Prandtl–Meyer flow indicated it varies from ray to ray.

The wedge angle is assumed to equal the angle formed by the vector V_{n_∞} with the line of intersection of the plane tangent to the wing surface and the plane perpendicular to it at the leading edge.

The gasdynamic functions on the rays are calculated from the velocity V_{n_∞}. The resultant velocity vector at the surface Q_1 is equal to the sum of the constant component V_{τ_∞} and the component obtained on the corresponding ray. By this means the necessary values of the gasdynamic functions are obtained for each layer at the boundary of region 3 formed by the surface Q_1.

Since the Prandtl–Meyer flow propagates from the leading edge of the wing within the stream up to the characteristic cone issuing from the wing tip, for economy in the calculation the surface Q_1 (in the particular case of a straight edge, Q_1 is a plane) should be located at the line of intersection of the characteristic cone with the wing surface.

The flow at the upper surface of the half-wing has a pronounced transverse velocity component w which develops in the expansion near the leading edge and is directed toward the axis of symmetry of the wing. At the plane of symmetry the components w arriving from the two halves of the wing are mutually cancelled, which results in the turning of the stream. The turning and compression of the stream near the plane of symmetry of the wind occur abruptly. The shock wave formed is small and does not produce large changes in the entropy. It is located within the characteristic cone issuing from the wing tip and in its lower part is normal to the wing surface. The existence of this shock wave has been noted earlier [4, 5] and confirmed by experiments [6].

3. Let us examine the problem of determining the initial data at the plane Q_0 for region 1. It is solved by the method of determining the self-similar expansion flow, just as for compression flow [2]. The algorithm of transition from layer to layer from the problem for region 3 is used repeatedly until self-similarity is established with respect to the coordinate t with the assigned accuracy. One can begin the determination from arbitrary data. For this purpose it is convenient to use the flow near two wedges: in the plane of symmetry and in the plane normal to the leading edge. Intermediate values at points can be obtained by interpolation. The accuracy of the determination can be controlled with respect to individual values or from graphs of the functions w and p, since these functions are established more slowly than the others. The quality of the solution and its determination can also be judged from the behavior of the entropy function $S = p/\rho^k$ which must be equal to unity everywhere in the stream (except the vicinity of the internal shock wave). Since the entropy function does not enter directly into the algorithm, it must be calculated separately for purposes of control.

The fact of the existence of a shock wave was not taken into account directly in the algorithm. The shock wave showed up in the results in a "diffuse" form. Usually four to five calculation points were located in the zone of the shock wave. Besides the "internal viscosity" (which occurs because the difference system is equivalent to the initial equations plus the approximation error) the introduction of an artificial "viscosity" with respect to the coordinate η with a small regulating parameter on it and a single smoothing of the functions at the layer contributed to an increase in the stability of the difference system in calculating the discontinuity. The problem for region 1 has an independent meaning for wings with a conical surface.

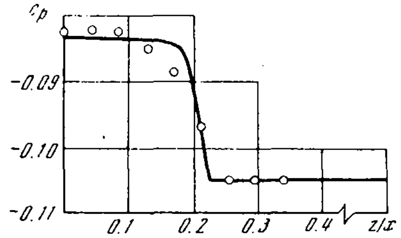

Fig. 4

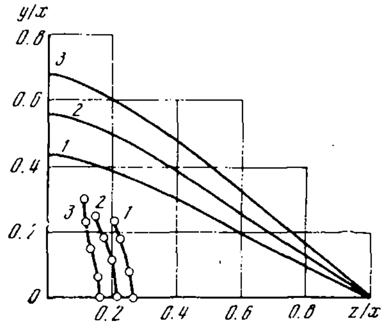

Fig. 5

4. Calculations of the flow over the upper surface of triangular plates with variation in M_∞, the sweepback angle χ of the leading edge, and the angle of attack α were conducted on an electronic computer according to the algorithm described.

In the calculations the quality of the solution was controlled with respect to the values of the entropy function and the value of the Bernoulli integral which does not enter into the algorithm. Everywhere except the region of the shock wave the entropy function differs by no more than 5% from unity, while the Bernoulli integral differs by no more than 1% from its value in the impinging stream.

The pressure at the upper surface of a triangular plate with $M_\infty = 6$, $\chi = 60°$, and $\alpha = 7°$ coincides with the calculated results from [5].

A comparison of the calculation with experiment [6] is shown in Fig. 2. In the experiment a special probe introduced into the stream measured the total pressure behind the direct shock wave formed in front of the probe in the field between the upper surface of a triangular plate and the outer characteristic surface. This pressure was divided by the total pressure behind the direct shock wave in the undisturbed stream. A plate with $\chi = 44.7°$ and $\alpha = 12°$ at $M_\infty = 2.94$ was used in the experiment. The experimentally obtained values of the pressure ratio $\tilde{p}$ over the span of the wing are shown in Fig. 2 by crosses and the calculated results by circles. The relative distance from the surface of the wing at which the pressure was measured along the span of the wing was $y/x = 0.1282$, and z/x are the half-spans divided by the base wing chord.

The position of the internal shock wave was determined in this experiment. The probable position of the projection of the shock wave on the plane x = const is shown in Fig. 3 by line segments. The position of the shock wave obtained from the calculation is plotted by circles. The calculated position of the trace of the characteristic surface on the plane x = const is also plotted.

Since the shock wave is "diffuse" in the calculations and the graph of pressure over the span does not have a clearly expressed "step," the position of the shock wave was determined from the pressure graphs as the point corresponding to half the sum of the maximum and minimum pressures at the step.

The intensity of the shock wave decreases along the direction from the wing surface toward the outer characteristic surface. At a certain distance from the wing surface the shock wave degenerates. This is indicated by the absence of a pressure drop on the graphs beginning with some value of the coordinate ξ.

A comparison of the present calculations (circles) with calculations of the pressure coefficient c_p at $M_\infty = 3$, $\chi = 45°$, and $\alpha = 12°$ from [7] (solid line) is presented in Fig. 4. A system of continuous calculation through the discontinuity by the "predictor–corrector" method with a larger number of calculated points than in the present work was used in [7]. The discontinuity is approximated well enough by the proposed system with a calculating grid of 9×9 points within the "core" of the stream (the core is bounded by the plane of symmetry and the plane Q_1 located on the line of intersection of the characteristic cone with the wing surface).

The traces on the plane x = const of the outer characteristic surfaces and the internal shock waves of a triangular plate with $M_\infty = 4$ and $\chi = 45°$ for different angles of attack are shown in Fig. 5. It is seen that as the angle of attack increases, the shock wave approaches the plane of symmetry of the wing. Here α = 5, 10, and 15° for 1, 2, and 3, respectively. An analysis of the change in the position of the shock wave on the upper surface of triangular plates having different degrees of sweepback of the leading edge shows that the sweepback has little effect on the position of the shock wave.

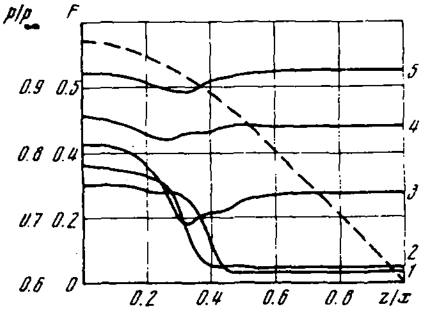

Fig. 6

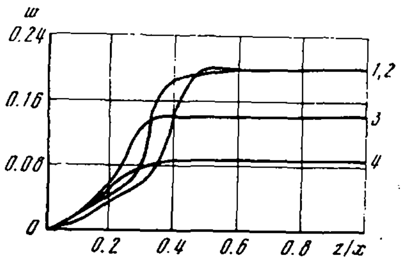

Fig. 7

Typical graphs of the variation in $p/p_\infty = p$ and $w = w/\sqrt{p_\infty/\rho_\infty}$ within the stream over the span of the upper surface of a triangular plate with $M_\infty = 2$, $\chi = 45°$, and $\alpha = 7°$ are presented in Figs. 6 and 7. In the graphs 1, 2, 3, 4, and 5 correspond to $\xi = 0, 0.5, 0.625, 0.75$, and 0.875. Here $\xi = y/F$, where F is the distance (in units of the base chord) from the surface of the plate to the outer characteristic surface. The position of this surface is shown by a dashed line in Fig. 6. It is seen from the graphs that with $\xi = 0.75$ the internal shock wave is already almost degenerated. This variant of the calculation lies at the limit of applicability of the present method of solution, since the turning of the stream in the plane normal to the leading edge is close to the limit for the lower surface of such a wing and consequently close to departure of the shock wave from the leading edge and violation of the statement of the problem.

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